

# Kinematic Analysis of Planar Biomechanical Models using Mixed Coordinates

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## EXTENDED ABSTRACT

### 1 Introduction

Kinematic analysis (KA) is a powerful tool used in the study of biomechanical systems, since it allows for the computation of the orientation of the model segments, trajectory of specific points, angular displacement of joints, among other variables of interest. Two approaches can be used to perform the kinematic analysis of multibody systems, namely, forward (FK) or inverse kinematics (IK). In the first case, the model is guided using linear and angular drivers calculated in a previous step. Afterwards, the consistent generalized coordinates are obtained by imposing the kinematic constraints that define the model. On the other hand, in IK the position and orientation of each segment is computed by minimizing the difference between the experimental data and a set of points belonging to the model, namely the coordinates of the system or other points of interest. This procedure allows for the fitting of the computational model to the experimental data.

In biomechanical models, FK should be applied with caution due to experimental errors associated to the measurement, in particular soft tissue artifacts (STA) [1]. The STA refers to the motion of the markers on the surface of the body with respect to the underlying bones due to inertial effects, skin deformation and sliding, gravity and muscle contraction [2]. Moreover, STA is task- and subject- dependent, which makes standard filtering techniques ineffective.

Andersen et al. (2009) showed that the use of methodologies to minimize the errors between experimental markers and model points result in significant differences in the kinematic outcomes when compared with standard methods. On its turn, these differences can lead to large errors and inconsistencies during dynamic analysis [3]. Consequently, a method that enables to adjust the model to the system in study is of particular interest for the biomechanics area, since it can minimize the errors associated to the experimental acquisition of anatomical points that constitute the biomechanical model. To address this issue, several methods have been proposed, being the most common based on optimization techniques [4].

In this work, a new approach based exclusively on kinematic constraints and least-square minimization is proposed to perform the kinematic analysis of biomechanical systems. The methodology considers the use of angular coordinates to model the kinematic drivers of the system. These coordinates are referred to as 'mixed coordinates' and complement the set generalized coordinates used by the Fully Cartesian Coordinates (FCC) formulation adopted [5]. This method enables to perform an IK analysis and to determine simultaneously the angular drivers of the model. It allows also for the minimization of the error between experimental and computational points, ensuring a better fit of the model to the experimental data.

### 2 Methods

The mixed coordinates (MC) formulation is defined as a combination of FCC with generalized angular coordinates. These coordinates represent the angular degrees-of-freedom of the kinematic pairs of the model, which will be calculated during the IK analysis. Therefore, MC allow for the simultaneous computation of the generalized coordinates of the biomechanical model and its joint angles.

However, this approach leads to an augmented vector of generalized coordinates of the system, since a new vector with length equal to the total number of angular degrees of freedom of the system is appended to the already existent vector of generalized coordinates. Thus, the use of additional kinematic constraint equations, which will be introduced in the form of trajectory constraints, is required. These trajectory drivers will map the experimental coordinates of points of interest of the model.

An important aspect of MC is that it only requires a change in the structure of the angular kinematic constraint equations of the FCC formulation, since the angle between the vectors of the bodies become a generalized coordinate of the system. Hence, its contribution to the Jacobian matrix of the system is different from the one in FCC, as it includes the terms dependent of the angular coordinates.

The MC were applied in the analysis of 3 gait cycles of a healthy female adult. Kinematic data were collected in the Lisbon Biomechanics Laboratory at Instituto Superior Técnico using 14 infrared Qualisys cameras with a sampling frequency of 100 Hz. The acquisition protocol was based on the PlugInGait model. The location of the hip joints was determined based on regression equations [6], whereas ankle, knee, elbow, and wrist joints were calculated based on the coordinates of the respective lateral and medial markers.

To compare the error associated with each approach, namely, FCC and FCC+MC, the root mean square errors (RMSE) between the experimental coordinates of each joint center and its estimation based on the consistent generalized coordinates of the model were computed. Additionally, the CPU time required to perform each kinematic analysis was also measured.

### 3 Results

The errors associated with the position of the joints were lower in the IK analysis with the MC formulation. On average, both methodologies present identical accuracy. A similar trend was observed for the maximum error between the IKA outputs and the FKA with angular drivers. Regarding the CPU times, the FK analysis with FCC took 4.49s and approximately 4 iterations per time frame to obtain the solution using the Newton Raphson method. On the other hand, the IK analysis with MC required 12.95s and an average of 8 iterations. These differences may be explained by the higher dimension of the Jacobian matrix in the MC formulation (FK:  $\Phi_{q[58 \times 48]}$ , IK:  $\Phi_{q[82 \times 60]}$ ). However, it is important to note that the processing time spent to obtain the initial angular drivers in the FK case was not included, while in the IK approach the time already considers all the required steps to perform the kinematic analysis and obtain the joint angles and generalized coordinates of the system.

Table 1: Root Mean Square Error (RMSE) between the experimental coordinates of each joint center and its estimation based on the consistent generalized coordinates of the model

Formulation	RMSE (mm)												
	Neck	Shoulder		Elbow		Wrist		Hip		Knee		Ankle	
	-	Right	Left	Right	Left	Right	Left	Right	Left	Left	Right	Left	Right
FCC	5.7	9.0	8.7	8.2	9.2	13.5	15.3	14.5	14.6	24.6	24.8	20.9	21.9
FCC + MC	0.005	0.007	0.007	0.005	0.005	0.004	0.004	0.009	0.004	0.006	0.006	0.015	0.017

### 4 Discussion

In general, the accuracy of the kinematic reconstruction using the FCC with MC is significantly higher than when only FCC are used. This issue is the direct result of the minimization of the distance between the model points and experimental data introduced by the method. Moreover, the IK analysis considering the FCC+MC formulation presents the advantage of computing the angular drivers that rule the system, without a preprocessing step.

The use of IK analysis enables also to reduce some of the experimental errors introduced by the use of markers to track the human body, namely the SMA. By simultaneously minimizing the distance to all joints, the method finds a position that better depict the experimental movement, correcting possible displacements of the markers. In addition, this method avoids one of the main drawbacks of the FK analysis, namely the propagation of errors along the kinematic chain, i.e., experimental errors will be passed continuously to the child bodies, resulting, in general, in higher distances between the experimental and model points in the distal joints.

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